

A certain stop light is programmed to take traffic conditions into consideration. A driver who pulls up to the light **SCORE: ____ / 25 PTS** when it is red will have to wait between 1 and 27 seconds. Let X be the amount of time (in seconds) that a random driver must wait for a red light to turn. Find the average (mean) time that a random driver has to wait if the probability density function of X has the form

$$f(x) = \begin{cases} \frac{k}{\sqrt[3]{x}}, & x \in [1, 27] \\ 0, & x \notin [1, 27] \end{cases}$$

NOTE: Your final answer must be a number.

$$\int_1^{27} kx^{-\frac{1}{3}} dx = 1 \quad (4)$$

$$(4) \quad \frac{3}{2} kx^{\frac{2}{3}} \Big|_1^{27} = 1$$

$$\frac{3}{2} k(9-1) = 1$$

$$(4) \quad k = \frac{1}{12}$$

$$\int_1^{27} \frac{1}{12} x \cdot x^{-\frac{1}{3}} dx \quad (5)$$

$$= \int_1^{27} \frac{1}{12} x^{\frac{2}{3}} dx$$

$$= \frac{1}{12} \cdot \frac{3}{5} x^{\frac{5}{3}} \Big|_1^{27} \quad (4)$$

$$= \frac{1}{20} (243 - 1)$$

$$= \frac{242}{20}$$

$$= \frac{121}{10} (4) = 12.1 \text{ SECONDS}$$

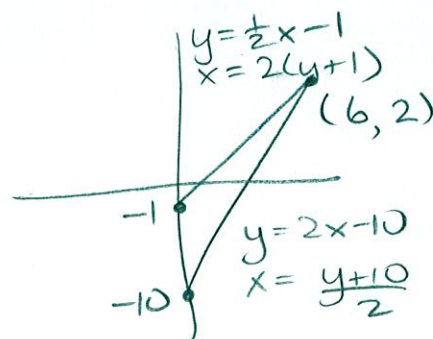
Consider the region bounded by $y = 2x - 10$, $y = \frac{1}{2}x - 1$ and $x = 0$.

SCORE: ____ / 35 PTS

[a] Write, **BUT DO NOT EVALUATE**, a dy integral (or sum of integrals) for the volume if the region is revolved around the line $x = -3$.

$$\pi \int_{-10}^{-1} \left[\left(\frac{y+10}{2} + 3 \right)^2 - (3)^2 \right] dy \quad (1)$$

$$+ \pi \int_{-1}^2 \left[\left(\frac{y+10}{2} + 3 \right)^2 - (2(y+1) + 3)^2 \right] dy \quad (1)$$



[b] Write, **BUT DO NOT EVALUATE**, a **single** integral for the volume if the region is revolved around the line $x = 8$.

$$2\pi \int_0^6 (8-x) \left(\frac{1}{2}x - 1 - (2x - 10) \right) dx$$

Find the length of the curve $y = \frac{1}{3}(x-3)\sqrt{x}$ over $4 \leq x \leq 16$.

SCORE: ____ / 25 PTS

NOTE: Your final answer must be a number.

$$y = \frac{1}{3}x^{\frac{3}{2}} - x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}} \quad (4)$$

$$\textcircled{2} \int_4^{16} \sqrt{1 + \left(\frac{1}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}\right)^2} dx \quad \textcircled{2} \leftarrow \text{MUST BE PRESENT IN ALL } \int$$

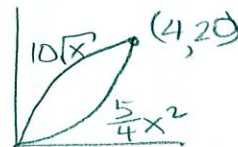
$$= \int_4^{16} \sqrt{1 + \frac{1}{4}x - \frac{1}{2} + \frac{1}{4}x^{-1}} dx \quad \textcircled{3}$$

$$= \int_4^{16} \sqrt{\frac{1}{4}x + \frac{1}{2} + \frac{1}{4}x^{-1}} dx \quad \textcircled{3}$$

$$= \int_4^{16} \left(\frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\right) dx \quad \textcircled{3}$$

$$= \left(\frac{1}{3}x^{\frac{3}{2}} + x^{\frac{1}{2}}\right) \Big|_4^{16} = \frac{1}{3}(64-8) + (4-2) = \frac{56}{3} + 2 = \frac{62}{3} \quad \textcircled{2}$$

Find the centroid (center of mass) of the region bounded by $y = 10\sqrt{x}$ and $y = \frac{5}{4}x^2$.



SCORE: ____ / 25 PTS

NOTE: Your final answer must be numeric.

$$\int_0^4 (10x^{\frac{1}{2}} - \frac{5}{4}x^2) dx = \left(\frac{20}{3}x^{\frac{3}{2}} - \frac{5}{12}x^3\right) \Big|_0^4 = \frac{20}{3} \cdot 8 - \frac{5}{12} \cdot 64 = \frac{160-80}{3} = \frac{80}{3} \quad \textcircled{3}$$

$$\int_0^4 x(10x^{\frac{1}{2}} - \frac{5}{4}x^2) dx = \int_0^4 (10x^{\frac{3}{2}} - \frac{5}{4}x^3) dx = \left(4x^{\frac{5}{2}} - \frac{5}{16}x^4\right) \Big|_0^4 \quad \textcircled{3}$$

$$= 4 \cdot 32 - \frac{5}{16} \cdot 4^4 = 128 - 80 = 48 \quad \textcircled{2}$$

$$\frac{1}{2} \int_0^4 ((10x^{\frac{1}{2}})^2 - (\frac{5}{4}x^2)^2) dx = \frac{1}{2} \int_0^4 (100x - \frac{25}{16}x^4) dx \quad \textcircled{2}$$

⓫ IF MISSING ANY dx
(1 DEDUCTION ONLY)

$$\begin{aligned} &= \frac{1}{2} \left(50x^2 - \frac{5}{16}x^5\right) \Big|_0^4 \quad \textcircled{3} \\ &= \frac{1}{2} (50 \cdot 16 - \frac{5}{16} \cdot 4^5) \\ &= \frac{1}{2} (800 - 320) = 240 \quad \textcircled{2} \end{aligned}$$

$$\frac{1}{\frac{80}{3}} (48, 240) = \frac{3}{80} (48, 240) = \left(\frac{9}{5}, 9\right) \quad \textcircled{2}$$

A solid of revolution has volume $\int_1^4 2\pi(e^y + y)(6 - y) dy$. Sketch the region and the axis of revolution.

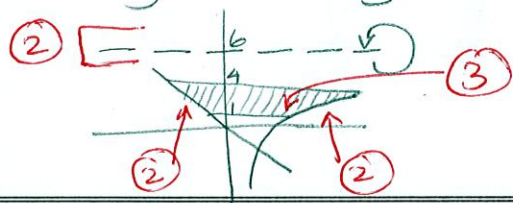
SCORE: ____ / 15 PTS

Do not use the x- nor y-axes as boundaries nor the axis of revolution.

SHELL METHOD, HORIZONTAL CUT // HORIZONTAL AXIS OF REVOLUTION

$$r = 6 - y \quad \text{AXIS } y = 6 \quad (2)$$

$$h = e^y + y = e^y - (-y) \quad \text{BOUNDARIES } \begin{matrix} x = e^y \rightarrow y = \ln x \\ x = -y \rightarrow y = -x \end{matrix} \quad (1) \quad (1)$$



Find the area of the region between the curves $y = \cos x$ and $y = \sin 2x$ over $0 \leq x \leq \frac{\pi}{2}$.

SCORE: ____ / 25 PTS

NOTE: Your final answer must be a number.

$$\cos x = \sin 2x$$

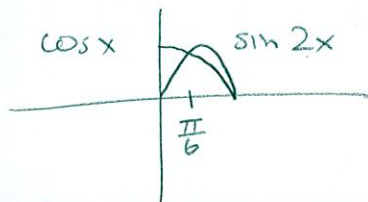
$$\cos x = 2 \sin x \cos x$$

$$0 = 2 \sin x \cos x - \cos x$$

$$= (2 \sin x - 1) \cos x$$

$$2 \sin x - 1 = 0 \rightarrow \sin x = \frac{1}{2} \rightarrow x = \frac{\pi}{6}$$

$$\cos x = 0 \rightarrow x = \frac{\pi}{2}$$



$$\begin{aligned} & \int_0^{\frac{\pi}{6}} (\cos x - \sin 2x) dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin 2x - \cos x) dx \\ &= \left(\sin x + \frac{1}{2} \cos 2x \right) \Big|_0^{\frac{\pi}{6}} + \left(-\frac{1}{2} \cos 2x - \sin x \right) \Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} \\ &= \left[\left(\frac{1}{2} + \frac{1}{4} \right) - \left(0 + \frac{1}{2} \right) \right] + \left[\left(\frac{1}{2} - 1 \right) - \left(-\frac{1}{4} - \frac{1}{2} \right) \right] \\ &= \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \end{aligned}$$